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ABSTRACT

This study investigated three factors associated with third grade children's ability to solve open sentences of the types N + a = b, a + N = b, a - N = b, and N - a = b, where a and b are given whole numbers and N is to be found. Two factors were: (1) the size of the numbers a and b; and (2) the context of the sentence, whether presented alone or as a step in the solution of a verbal problem. A 16-item test was constructed by completely crossing sentence type with these two factors, and administered to a random sample of 16 boys and 16 girls drawn from three representative elementary schools. A latin square procedure was used to control any item sequence effects, and every child received the same 16 items in a different order. Response scores, analyzed by MANOVA, showed that: (1) there was no significant sex effect; (2) the sentence type a - N = b was significantly more difficult than the other types, which did not differ significantly; (3) sentences with one-digit numbers were significantly easier than those with two digit numbers; and (4) the verbal problems used in the test were not more difficult than the corresponding abstract number sentences. (MM)

SOME FACTORS ASSOCIATED WITH CHILDREN'S SOLVING PERFORMANCE ON FOUR TYPES OF MATHEMATICAL OPEN SENTENCES

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ABSTRACT

In this study the relationship between children's performance in solving certain open sentences and three factors (Open Sentence Type, Number Size, Context) associated with these open sentences was investigated. Sixteen open sentence solving tasks were individually presented to each of thirty-two randomly selected third grade children.

Multivariate analysis of variance on the number of correct solutions on linear combinations of solving tasks indicated that there were significant differences in children's solving performance on the four types of open sentences and on the two levels of number size. There also was a significant Open Sentence Type X Number Size Interaction.

PROBLEM, RATIONALE, AND BACKGROUND

The purpose of this study was to investigate the relationship between children's performance in solving certain open sentences and three factors (Open Sentence Type, Number Size, Context) associated with these open sentences. A distinguishing feature of contemporary mathematics programs is the prominence of open sentences at every grade level. In the primary grades children regularly encounter open sentences such as $7 + 2 = \square$, $8 + \square = 15$, $9 - 6 = \square$, and others. Much of this experience with open sentences is associated with the development of computation or problem solving skills. In spite of their importance in the mathematics curriculum very little research has focused on the many important facets of open sentences. Consequently, many curricular and instructional decisions have not been made with the aid of sufficient research evidence. The aim of this study was to begin to alleviate this situation by answering several questions associated with third grade childrens performance in solving open sentences. The questions centered around the relationship between solving performance (i.e., number of correct solutions) and the type of open sentence being solved, the size of the numbers involved, and the presence of a verbal problem. Each of these relationships will now be discussed in more detail.

Open Sentence Types

In examining the research literature it quickly became apparent that research pertaining to children's performance levels on different types of open sentences was needed. Other than the classical studies concerned with the relative difficulty of the basic facts very few studies have been reported on this issue. For an overview and references concerning this literature see Grouws (1971).

The variety of open sentences which appear in mathematics, even when restricted to school mathematics programs, is very large. Therefore, to keep the investigation within manageable size the kinds of open sentences studied were delimited.

The open sentences considered are commonly referred to in mathematics as first degree equations in one variable. In particular, the four open sentence types studied in this investigation were: $N + a = b$, $a + N = b$, $N - a = b$, and $a - N = b$ where in each type, N was a placeholder for a number, and $\underline{a}$ and $\underline{b}$ were whole numbers. Table I lists these four open sentence types and the labels used to refer to each.

Table 1

The Four Types of Open Sentences
Studied in this Investigation

	Labels			
	Type 1	Type 2	Type 3	Type 4
Open Sentence Types	N + a = b	a + N = b	a - N = b	N - a = b

Note -- N is a placeholder for a number and a and b are whole numbers.

The rationale for selecting these open sentence types was based on four considerations. First, equations involving variables are important mathematically and socially. Second, first degree equations in one variable involving addition and subtraction are basic members of this set of equations, and are the equations given the first broad coverage in contemporary mathematics programs. Third, first degree equations in one variable involving addition and subtraction where the particular operation specified occurs in the left member of the equation (e.g., 4 + N = 9) are much more common in school mathematics programs than similar equations where the operation is specified in the right member of the equation (e.g., 8 = 5 + N). Consequently, only "operation left" equations were considered since some familiarity with the open sentences used was deemed desirable.

Fourth, solving equations in canonical form, that is, where the placeholder occurs by itself in one member of the equation (e.g., 14 + 37 =N), involves straight-forward processing of numerals (i.e., use of traditional algorithms) to obtain a standard name, and is frequently mastered far in advance of the ability to solve equations not in this form (e.g., 14 + N = 49). Since methods of solution were of concern, in an aspect of the investigation not being reported here, equations in canonical form were excluded from the study.

Number Size

Is 9 + N = 16 easier to solve than 39 + N = 86? To what extent? What part do computation errors play in any differences that exist? Answers to questions such as these were deemed of interest since they have important implications. It was decided, therefore, to examine the relationship between solving performance and the magnitude of the numbers in the open sentences being solved. Only whole numbers were studied in this investigation. The decision to use whole numbers was based on the need to keep the size of the study within manageable bounds and the following assertions. First, whole numbers are certainly one of the basic sets of numbers designated for study in contemporary mathematics curricula. Therefore, consideration of these numbers was important. Second, the whole numbers are the set of numbers initially used in school mathematics programs in connection with the restricted set of open sentences studied in this research. Thus, to minimize transfer of training from any previous work with open sentences the set of whole numbers was a natural choice.

Clearly, the exclusive use of whole numbers as constants provided neither a means of determining the influence of number size on solving performance nor a control for the magnitude of the whole numbers used when examining other factors. To solve the first part of this problem two disjoint subsets of the set of whole numbers were selected and performance associated with each eventually compared. These subsets were chosen to meet certain criteria. The first criterion was that the whole numbers in each of the subsets be familiar to the third-grade children involved.

The second criterion was that the children had experience in finding sums and differences of numbers of the same magnitude as those in each of the subsets chosen. The third criterion was that the subsets of the whole numbers chosen must allow, to a reasonable extent, the factor of number size (magnitude) to operate.

On the basis of the preceding criteria two disjoint subsets were identified. One of these subsets, called the Basic Fact domain, was composed of whole numbers less than 19. The other subset called the Two-digit domain included whole numbers greater than 20 but less than 99.

Obviously, each whole number in the basic fact domain was of smaller magnitude than any whole number in the Two digit domain. Also, the constants in any open sentence used in the investigation were chosen from only one domain. Therefore, as a result of this systematic choice of number domains evidence pertaining to the influence of number size was obtained. Specifically, data were generated on differences in children's solving performance on open sentences due to the magnitude of the whole numbers in the number domain from which the constants were selected. Finally, the method used to control for number size in determining the relationship (or lack of relationship) of other factors to solving performance is discussed later.

Context

Frequently open sentences are used as part of a verbal problem solving technique. The usual procedure involves writing an open sentence which models the mathematical conditions expressed in the verbal problem. The solution set of this sentence is then determined and related back to the given problem in expressing the answer. A question investigated was whether solving an open sentence was related to the presence or absence of a verbal problem. This factor, pertaining to the influence of a verbal problem on the solving of a related open sentence, was called the context factor and had two levels. They were called the verbal-symbolic context and the symbolic context. The essential difference between these two levels was that in the former an appropriate verbal problem was presented with each open sentence to be solved and in the later, verbal problems were excluded from the open sentence solving tasks.

No research on the influence of verbal problems on the solving of open sentences was found in the literature. Since only one kind of verbal problems was used in the study this facet of the investigation is primarily exploratory in nature. Exact specifications concerning the verbal problems used are stated later.

Interactions Among Factors

Also of interest in the study were questions concerning interactions among the three factors just discussed. For example, are children's solving performance levels on the four types of open sentences studied, similar within each of the levels of the number size factor? One aim of the study was to find an answer to this question and other questions related to interactions among the three factors.

The Study

Specifications for 16 test items, each involving an open sentence, were developed so that children's performance associated with the factors of interest could be systematically measured. Details concerning the experimental design, the sample, the test, the test administration procedure, the data, and the statistics are presented in the following parts.

The Experimental Design

The design of the study with respect to subjects was a 2 X 2 factorial design. Sex and two levels of Order of Presentation make up the dimensions of this design. With respect to the sixteen repeated measures, these form a 4 X 2 X 2 design given by four types of open sentences, two number sizes, and two contexts. All factors in these designs are fixed. A representation of the design is given in Table 2.

The Sample

Thirty-two third-grade children selected from the Madison (Wisconsin) Public Schools formed the sample for this study. Three elementary schools from the city's thirty-four elementary schools were identified as representative of the elementary schools of the district by the mathematics consultant of the district. Sixteen boys and sixteen girls were randomly selected from the third-grade children attending the three selected schools. The order of selection was recorded and used later in the study.

The Test

Each of the thirty-two children in the sample was given sixteen problem situations. The same sixteen problem situations were given to each child. A problem situation was an open sentence, or an open sentence and a verbal problem. These problem situations were completely crossed with respect to the factors of: open sentence type (4 levels), number size (2 levels), and context (2 levels) (see Table 3). The problem situations were also balanced with regard to this crossing; that is, exactly one problem situation was generated from each cell of a 4 X 2 X 2 matrix where the four rows of the matrix represented the levels of the open sentence type factor, the two columns the levels of the number size factor, and the layers the levels of the context factor (see figure 1).

Table 2

Representation of the Experimental Design

Sex [illegible]	Order	T_1				T_2				T_3				T_4			
		C_1		C_2		C_1		C_2		C_1		C_2		C_1		C_2	
		N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2
Male	I																
	II																
Female	I																
	II																

Note. -- T_i ($i = 1,2,3,4$) represents the four open sentence types. C_i ($i=1,2$) represents the two problem situation contexts. N_i ($i = 1,2$) represents the two number sizes.

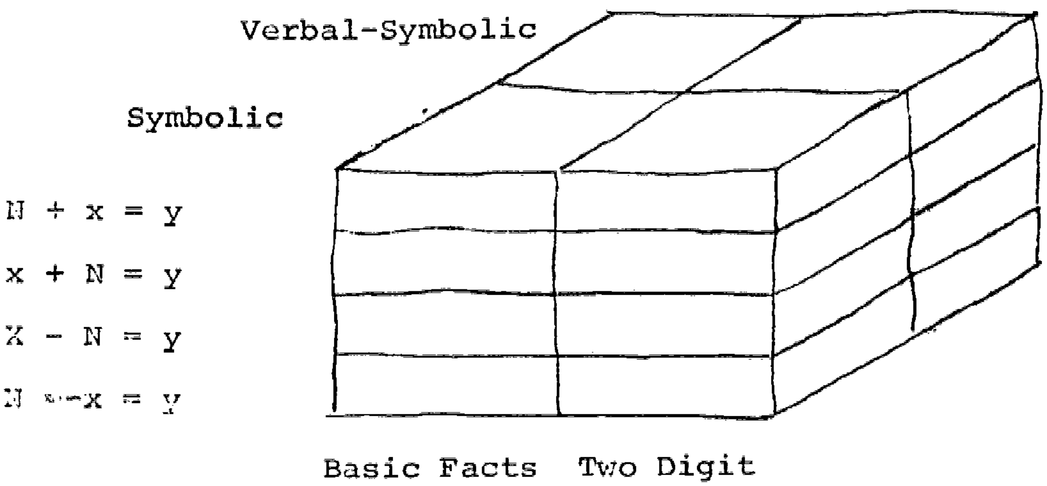
Table 4

Classification of Problem Situations by Context, Number Domain, and Open Sentence Type

Problem Situations	Factors		
	Context	Number Domain	Open Sentence Type
A	Symbolic	Basic Fact	I
B	Symbolic	Basic Fact	II
C	Symbolic	Basic Fact	III
D	Symbolic	Basic Fact	IV
E	Symbolic	Two Digit	I
F	Symbolic	Two Digit	II
G	Symbolic	Two Digit	III
H	Symbolic	Two Digit	IV
I	Verbal-Symbolic	Basic Fact	I
J	Verbal-Symbolic	Basic Fact	II
K	Verbal-Symbolic	Basic Fact	III
L	Verbal-Symbolic	Basic Fact	IV
M	Verbal-Symbolic	Two Digit	I
N	Verbal-Symbolic	Two Digit	II
O	Verbal-Symbolic	Two Digit	III
P	Verbal-Symbolic	Two Digit	IV

Figure 1

Representation of the Problem Situations



Note: N is a placeholder and x and y are whole numbers.

A problem situation in the symbolic context was an open sentence, A problem situation in the verbal-symbolic context was composed of an open sentence and a verbal problem. The open sentence used in a particular problem situation reflected the open sentence type and the number size associated with the given problem situation. For example, problem situation E (see Table 4) was a type I open sentence composed of numbers from the Two-digit number domain. The systematic methods used to generate open sentences and verbal problems appropriate to given problem situations is described later.

Although each child in the sample received the same 16 problem situations (i.e., A, B, C, D, ..., P) the order of presentation was not identical across children. The order of presentation was such that simple order effects due to the context factor were controlled. Although no item sequence effects were anticipated,as a safeguard latin squares were used to counterbalance for any such effects. Thus, each problem situation appeared exactly twice in each of the sixteen item positions.

Two number domains from which open sentences were generated were chosen to represent the two levels of the number size factor. The first domain, called the Basic Fact domain, consisted of ordered triples defined as follows:[1] $\{(a,b,c) \mid a,b,c \in \text{Counting Numbers}, a + b = c, a \neq b, 11 \leq c \leq 18, 1 \leq a \leq 9, 1 \leq b \leq 9\}$. From the thirty-two triples in this domain eight triples were randomly selected without replacement for use in this study. The thirty-two triples in this domain and the eight triples selected from it are defined in Table 5. The second number domain called

[1] Note that the domains are composed of ordered triples of whole numbers rather than a subset of the set of whole numbers as previously indicated. Use of ordered triples facilitated systematic item construction.

the Two-digit domain, consisted of ordered triples defined as follows: $\{(a,b,c) \mid a,b,c \varepsilon$ Counting Numbers, $a + b = c$, $a \neq b$, $42 \leq c \leq 99$, $21 \leq a \leq 78$, $21 \leq b \leq 78$, and the sum of the units digits of a and b is greater than $9\}$. From the 660 triples in this domain eight triples were randomly selected without replacement for use in this study. These eight triples are displayed in Table 6.

Open Sentence Generation

Given a triple from either number domain, say (a,b,c), then the Type I open sentence generated from this triple for use in the study was $N + b = c$ (henceforth, N is used as a placeholder for a number), the

Table 5

Number Triples in the Basic Facts Domain

(2,9,11)	(5,8,13)	(7,6,13)*	(8,9,17)*
(3,8,11)*	(5,9,14)	(7,8,15)	(9,2,11)
(3,9,12)	(6,5,11)	(7,9,16)	(9,3,12)
(4,7,11)*	(6,7,13)	(8,3,11)	(9,4,13)
(4,8,12)	(6,8,14)	(8,4,12)	(9,5,14)
(4,9,13)*	(6,9,15)*	(8,5,13)*	(9,6,15)
(5,9,14)	(7,4,11)	(8,6,14)	(9,7,16)
(5,7,12)	(7,5,12)	(8,7,15)	(9,8,17)*

*Number Triples randomly chosen from this domain for use in the study.

Table 6

Two-digit Domain Triples used in the Study

(39,21,60)	(62,28,90)	(66,27,93)	(71,19,90)
(24,39,63)	(58,35,93)	(55,36,91)	(37,54,91)

Type II open sentence was $a + N = c$, the Type III open sentence was $c - N = a$. For example, given the ordered triple (39,21,60) then the Type I open sentence generated from this triple was $N + 21 = 60$, the Type II open sentence was $39 + N = 60$, the Type III open sentence was $60 - N = 39$, and the Type IV open sentence was $N - 21 = 39$.

In order to generate verbal problems to be associated with the open sentences considered in this study an algorithm was developed such that given an open sentence type, an ordered triple, a proper name, and a name for a set of concrete objects, then a well-defined verbal problem unique to these four inputs was easily written. This was accomplished by constructing a verbal problem form for each of the four open sentence types (see Table 7). The procedure for constructing appropriate verbal problems thus becomes a simple substitution process. For example, given the open sentence type $N + x = y$ then the associated verbal problem form is as follows:

> You have some (objects). (Name) gives you (x) (objects). You now have (y) (objects). How many (objects) did (Name) give to you?

Also, given (4,7,11), Judy, and stamps as the other three inputs then by substitution the unique verbal problem determined is as follows:

> You have some stamps. Judy gives you 7 stamps. You now have 11 stamps. How many stamps did Judy give to you?

In summary, open sentence and verbal problem generation involved processes which were systematic in nature allowing for random selection and assignment wherever possible. For further details concerning test construction see Grouws (1971).

Table 7

Verbal Problem Forms for Each Open Sentence Type

Sentence	Verbal Problem Form	Given: Example Given: (4,7,11), Judy, Stamps
$x + N = y$	You have (x) (objects). (Name) gives you some (objects). You now have (y) (objects). How many (objects) did (Name) give to you?	You have 4 stamps. Judy gives you some stamps. You now have 11 stamps. How many stamps did Judy give to you?
$N + x = y$	You have some (objects). (Name) gives you (x) (objects). You now have (y) (objects) How many (objects) did (Name) give to you?	You have some stamps. Judy gives you 7 stamps You now have 11 stamps. How many stamps did Judy give to you?
$N - x = y$	You have some (objects). You give (x) (objects) to (Name). You now have (y) (objects). How many (objects) did you have to begin with?	You have some stamps. You give 7 stamps to Judy. You now have 4 stamps. How many stamps did you have to begin with?
$x - N = y$	You have (x) (objects). You give some (objects) to (Name). You now have (y) (objects). How many (objects) did you give to (Name)?	You have 11 stamps. You give some stamps to Judy. You now have 4 stamps. How many stamps did you give to Judy?

Test Administration

Each of the thirty-two tests was administered in two parts. Part I consisted of the first eight problem situations on any given test and Part II consisted of the remaining eight problem situations. Part I testing was done during the first seven days of April 1971 and Part II testing was completed during the period April 19-22, 1971. All testing was done on an individual basis by this researcher in a room apart from the child's regular classroom. Only the child and the interviewer were present during the testing. Testing time was approximately 15 minutes on each part. Each child was given as much time as he wanted on each item.

The Statistics

The numerical responses recorded on the Interview Coding Form were scored after the completion of testing. A score of 1 was assigned to those situations where the child's numerical response was the correct solution of the open sentence involved. All other situations were given a score of 0. The data on the numerical responses were computer analyzed (Finn, 1967) using appropriate MANOVA procedures which take account of the fixed model and the repeated measures. Differences in means associated with the Order of Presentation variable were tested using a t-test for differences between mean means associated with independent samples. The .05 level of significance was used in all decision making.

RESULTS

Table 8 lists the number of correct responses on each of the sixteen problem situations. The total number of correct responses was 286 of a possible total of 512 (32 children x 16 problem situations). The mean number of correct responses per child was 8.94 and the variance of the distribution of children's response scores was 14.90.

Table 8

Number of Correct Responses by Open Sentence Type, Context, Number Size, and Sex

Sex	T_1				T_2				T_3				T_4				Total Number Correct
	C_1		C_2		C_1		C_2		C_1		C_2		C_1		C_2		
	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2	
Male	12	5	15	4	16	6	16	7	15	6	13	5	6	4	8	3	141
Female	15	6	13	7	15	5	13	5	15	4	13	8	6	3	9	8	145
Total	27	11	28	11	31	11	29	12	30	10	26	13	12	7	17	11	286

Note. -- T_i (i = 1,2,3,4) represents the four open sentence types. C_i (i=1,2) represents the two contexts. N_i (i = 1,2) represents the two number sizes.

Statistical tests showed no significant difference between boys' and girls' solving performance on the 16 item test (see Tables 9 and 10). There also were no significant order effects.

Table 9

Total Response Score Data by Sex

Sex	N	Number of Correct Responses	Percent of Total Correct Responses (286)	Mean Correct Responses
Boys	16	144	50.3	9.00
Girls	16	142	49.6	8.88
Boys & Girls	32	286	100.0	8.94

Table 10

ANOVA on Total Response Scores
for a Sex Main Effect

Hypothesis	Error	MS	df	F-ratio	p less than
$\mu boys = \mu girls = 0$	.9620	.0078	(1,30)	.0081	.9288

<u>Open Sentence Type</u>

The sixteen test items were grouped according to the type of open sentence included. The number of test items in each of the four open sentence type groups was four. Information concerning solving performance on each open sentence type is summarized in Table 11.

Table 11

Summary of Data on Open Sentence Types

Open Sentence Type	Number of Items/Test	Total Number of Items	Number of Correct Responses	Mean Correct Responses
I	4	128	77	2.40
II	4	128	83	2.59
III	4	128	79	2.47
IV	4	128	47	1.47
Total	16	512	286	8.94

The total number of correct responses was 77 or 60.2 percent of the possible total of 128 (4 problem situations x 32 children) on problem situations involving Type I (N + a = b) open sentences, 83 or 64.8 percent on Type II (a + N = b), 79 or 61.7 percent on Type III (a - N = b), and 47 or 36.7 percent on Type IV (N - a = b). The mean number of correct responses was 2.40 on problem situations containing a Type I open sentence, 2.59 on Type II, 2.47 on Type III, and 1.47 on Type IV.

Since there were four levels of the open sentence type factor, three independent contrasts among means could be made. The results of the statistical tests related to the open sentence type factor are shown in Table 12. A multivariate test, which takes account of the correlation between measures, was performed to simultaneously test the nullity of the grand means of the three dependent measures. The results of this test (i.e., the obtained probability of the resulting test statistic) prompted consideration of the univariate hypotheses.

The first univariate test indicated that performance on test items involving Type I open sentences was not statistically different from the [illegible]age performance on test items involving Type II, Type III, or Type IV open sentences. The second univariate test indicated that performance on test items involving open sentences of Type II was statistically different from the average performance on test items involving Type III or Type IV open sentences. The number of correct responses on each open sentence type (see Table 11) indicated that the performance on Type II open sentences was better than the average performance across Types III and IV. The third univariate test indicated that performance on open sentences of Type III was different from the performance on open sentences of Type IV. Number of correct responses on problem situations associated with these types (see Table 11) indicated that the better performance was on Type III open sentences.

Number Size

Each of the sixteen items on the test was placed in one of two categories according to the magnitude of the constants in the associated open sentence. One category contained the eight problem situations involving open sentences with constants drawn from the Basic Fact domain. The other category contained the remaining eight problem situations which involved open sentences with constants drawn from the Two-digit domain. Table 13 summarizes data on the number size factor.

Table 13

Summary of Data on Number Size

Number Size	Number of Items/Test	Total Number of Items	Number of Correct Responses	Mean Correct Responses
Basic Facts	8	256	200	6.24
Two-digit	8	256	86	2.69
Total	16	512	286	8.94

Table 12

Summary of Tests on Open Sentence Types

Hypothesis	Error	MS	df	F-ratio	p less than
Multivariate			(3,28)	20.3801	.0001
$\mu(T_I) - 1/3\ \mu[T_{II} + T_{III} + T_{IV}] = 0$	.1297	.3151	(1,30)	2.4297	.1296
$\mu(T_{II}) - 1/2\mu[T_{III} + T_{IV}] = 0$	.0604	2.0833	(1,30)	34.4828	.0001
$\mu(T_{III}) - \mu(T_{IV}) = 0$	.1896	4.0000	(1,30)	21.0989	.0001

The total number of correct responses on problem situations in the Basic Facts category was 200 or 78.1 percent of the possible total of 256 (8 items x 32 children). For the problem situations in the Two-digit category the total number of correct responses was 86 or 33.6 percent of the possible total of 256. The mean number of correct responses was 6.25 and 2.69 for the eight Basic Fact items and the eight Two-digit items, respectively.

The two levels of the number size factor implied that only one independent contrast could be made. The results of the univariate test for a main effect (see Table 14) indicated that a difference in performance did exist, and examination of the number of correct responses (see Table 13) implied that the better performance was on the items involving open sentences with constants selected from the Basic Facts domain.

Context

The sixteen items (problem situations) on the test were divided into two categories according to whether a verbal problem was involved. The eight items in the symbolic context group did not involve a verbal problem, and the eight items in the verbal-symbolic context group did involve a verbal problem. Data related to the context factor are summarized in Table 15.

Table 14

ANOVA for a Number Size Main Effect

Hypothesis	Error	MS	df	F-ratio	p less than
$\mu(N_{BF}) - \mu(N_{T-d}) = 0$	.2536	25.3828	(1,30)	100.0719	.0001

Table 15

Summary of Data on the Context Factor

Context	Number of Items/Test	Total Number of Items	Number of Correct Responses	Mean Correct Responses
Symbolic	8	256	139	4.34
Verbal-Symbolic	8	256	147	4.59
Total	16	512	286	8.94

The total number of correct responses on the symbolic context problem situations was 139 or 54.3 percent of the possible total of 256 (8 items x 32 children). For verbal-symbolic context problem situations the total number of correct responses was 147 or 57.4 percent of the possible total of 256. The mean number of correct responses was 4.34 and 4.59 for the eight problem situations in each context, respectively.

The univariate test for a context main effect was not significant (see Table 16). Thus, mean performance on symbolic context problem situations was not statistically different from the mean performance on the verbal-symbolic context problem situations.

Table 16

ANOVA for a Context Main Effect

Hypothesis	Error	MS	df	F-ratio	p less than
$\mu(C_S) - \mu(C_{V-S}) = 0$	.1823	.1250	(1,30)	.6857	.4142

Interactions

The only interaction among factors that was statistically significant was the Open Sentence Type X Number Size interaction. The statistical tests on the contrasts associated with this interaction are summarized in Table 17.

Table 17

Summary of Tests on Open Sentence Type X Number Size Interaction

Hypothesis	Error	MS	df	F-ratio	p less than
Multivariate			(3,28)	3.6479	.0245
(1)	.0866	.2109	(1,30)	2.4349	.1292
(2)	.1858	1.1719	(1,30)	6.3084	.0177
(3)	.2281	1.8906	(1,30)	8.2877	.0073

The three contrasts investigated were:

(1) $\mu(S_{I,BF}) + 1/3 \quad \mu(S_{II,T-d} + S_{III,T-d} + S_{IV,T-d}) -$

$\mu(S_{I,T-d}) - 1/3 \quad \mu(S_{II,BF} + S_{III,BF} + S_{IV,BF}) = 0$

(2) $\mu(S_{II,BF}) + 1/2 \quad \mu(S_{III,BF} + S_{IV,T-d}) - \mu(S_{II,T-d}) - 1/2$

$\mu(S_{III,BF} + S_{IV,BF}) = 0$

(3) $1/2 \quad \mu(S_{III,BF} + S_{IV,T-d}) - 1/2 \, \mu(S_{III,T-d} + S_{IV,BF}) = 0$

where $S_{i,j}$ (i = I, II, III, IV; j = BF Basic Facts, T-d Two-digit) was performance over problem situations involving an i type open sentence whose constants were from the j number domain.

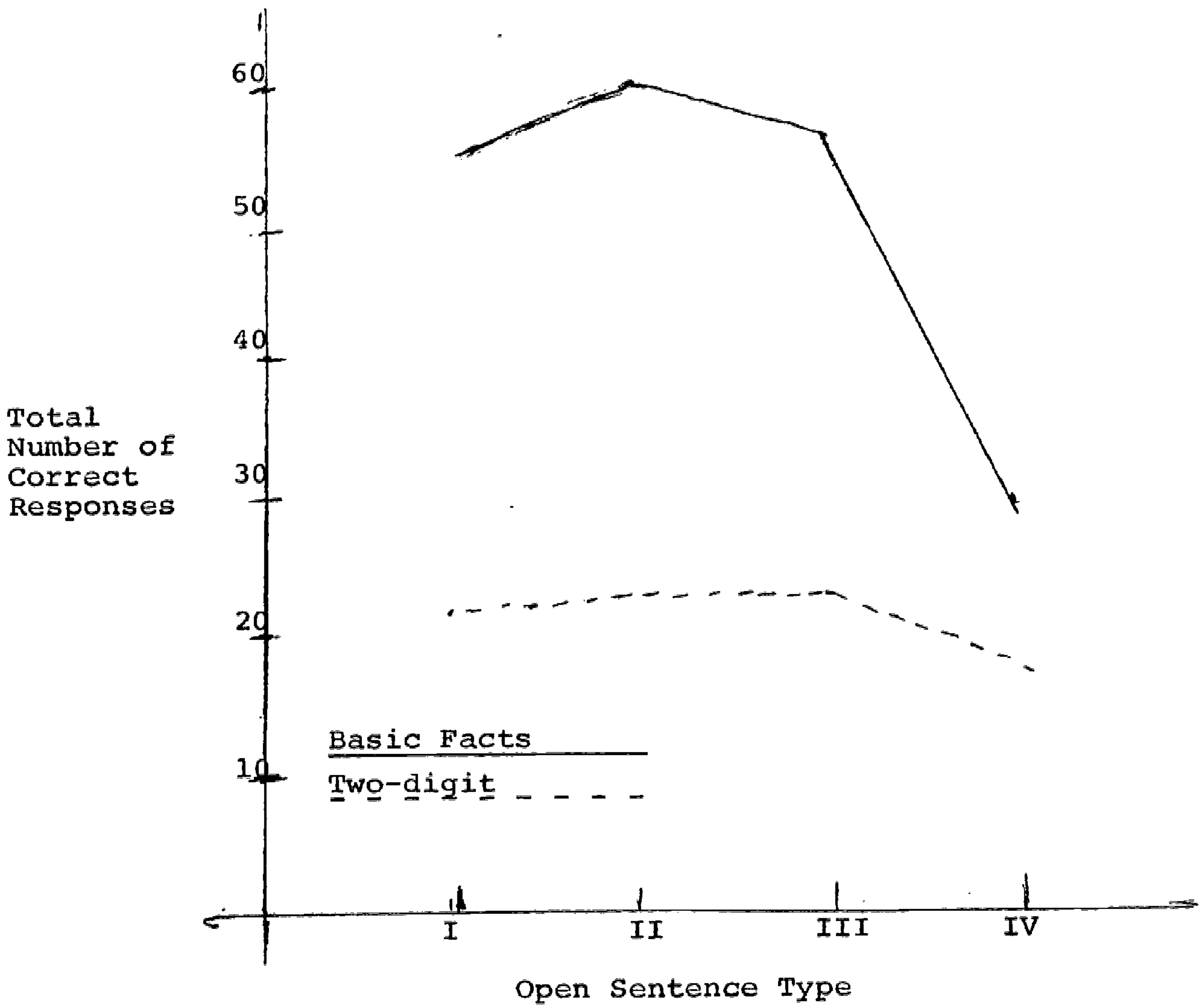
CONCLUSIONS

On the basis of the data collected and the statistical tests used there is no evidence to support the existence of a difference in boys' and girls' overall solving performance on open sentences of the kinds considered in this research. Also there is no reason to assume that the order in which open sentences are presented in testing situations influences children's performance in solving them.

Figure 2 is a graphic display illustrating the Open Sentence Type X Number Size Interaction.

Figure 2

Representation of Open Sentence Type X Number Size Interaction



Open Sentence Type

The results of the statistical tests previously reported support the conclusion that there are differences in children's solving performance on the four types of open sentences considered. These differences are illustrated in Figure 3 in terms of the number of correct responses associated with each open sentence type. Clearly, open sentences of the $N - a = b$ type are substantially more difficult than open sentences of the types $N + a - b$, $a + N = b$, and $a - N = b$. This also holds in each context (i.e., symbolic and verbal-symbolic) and each number domain (i.e., Basic Fact and Two-digit) although the difference between the $N - a = b$ type open sentence and each of the other three open sentence types is not as pronounced in situations involving Two-digit constants.

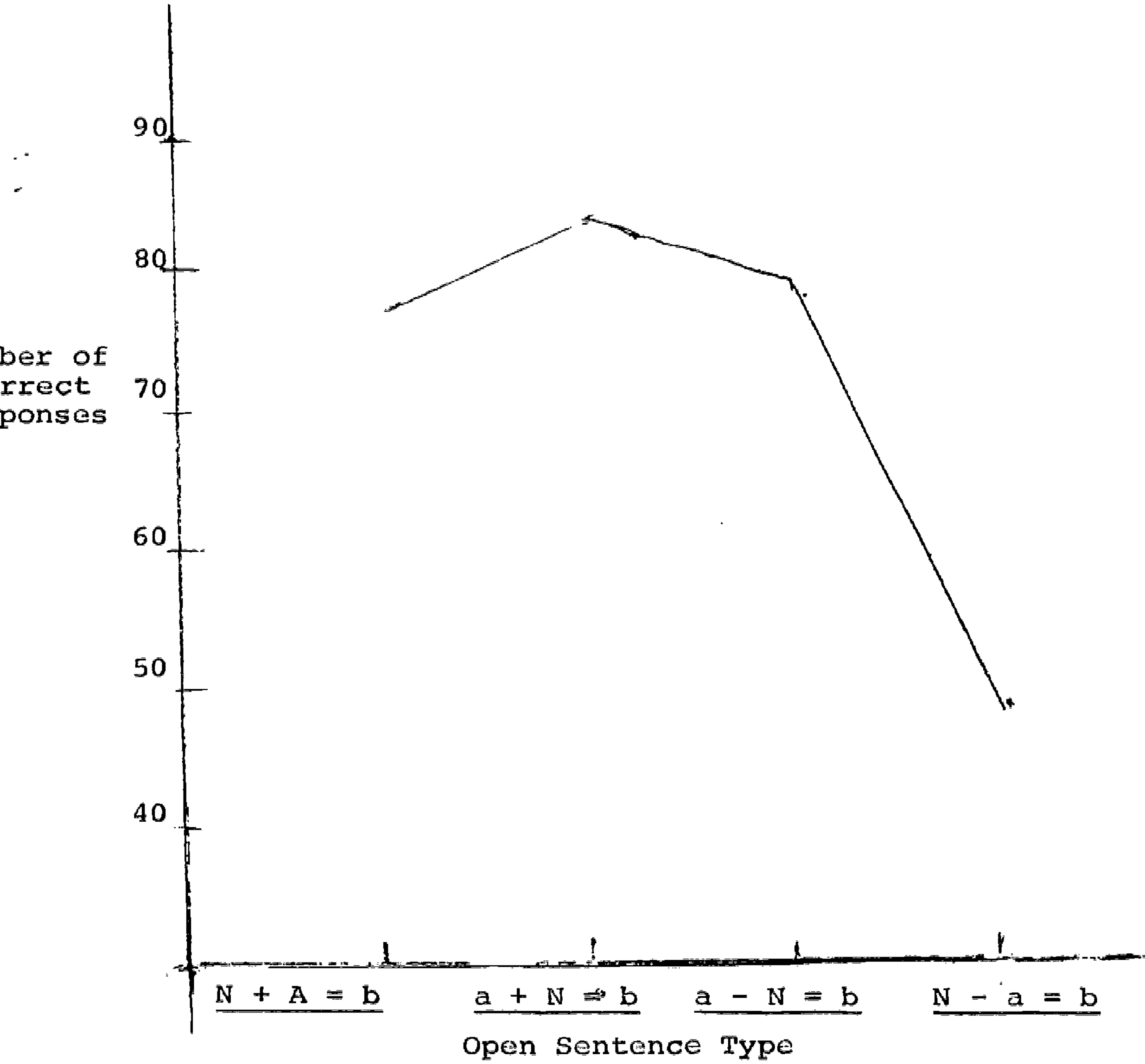
It should be noted that the part of the conclusion concerning the difficulty of $N + a = b$ and $N - a = b$ type open sentences is exactly opposite of the conclusion drawn by Suppes (Suppes et al., 1968, pp. 240-241). However, the data used to support Suppes' conclusion was not explicitly stated. Weaver's raw data[2] tends to confirm the conclusion reported here; namely, that $N - a = b$ type open sentences are more difficult to solve than $N + a = b$ type open sentences.

Three comments, each of a tentative nature, are in order. First, the evidence from the present study does not support Suppes' (Suppes et al., 1968) conclusion that there are substantial differences in the difficulty of open sentences of the type $N + a = b$ and the type $a + N = b$.

[2]Unpublished report made by Professor J. F. Weaver to the Madison (Wisconsin) Public Schools during the 1970-71 academic year.

Figure 3

Representation of Solving Performance on Open Sentences by Type



Second, examination of the data indicates that the four open sentence types considered in this study can be placed in order of difficulty by first considering placeholder position and then considering the operation involved. That is, the placeholder in the initial position (i.e., N - a = b or N + a = b) is the most difficult and within each of the two placeholder positions the open sentence type involving subtraction is most difficult.

Third, the relatively poor performance on Type IV open sentences (i.e., N - a = b) may reflect the relative amount of exposure children experience with respect to this type of open sentence as compared to other types of open sentences. Alternatively or concomitantly, this type of open sentence may be inherently more difficult to solve than the other three types of open sentences. That is, more time (or practice, or instruction, etc.) may be required for children to reach a certain proficiency in solving this type of open sentence than to reach the same degree of solving proficiency on the other three types of open sentences.

Number Size

From the results related to the number size factor it can be concluded that the magnitude of the whole numbers used as constants in open sentences of the types considered is related to children's solving performance. Specifically, open sentences with constants from the Two-digit domain are much more difficult for children to solve than open sentences with with constants from the Basic Fact domain.

As in the previous section this divergent performance may be attributable to opportunity to learn, or to specificity of learning, or both. However, in either case the assumption that children who can solve open sentences with small whole numbers as constants can also solve the same type of open sentence with whole numbers of larger magnitude as constants was not upheld. It is also interesting that the interviewer observed very few situations (i.e., less than 16) where a child did not correctly solve a "Two-digit open sentence" due to an error in the processing of a sum or difference using the traditional algorithms. Hence, it seems that not only are open sentences containing larger constants more difficult to solve, but the difficulty apparently involves something more than the computational complexity of processing larger numbers.

Context

From the results previously reported it may be concluded that the presence or absence of a verbal problem did not influence children's solving performance. At least two important things must be kept in mind with regard to this conclusion. First, only a very restricted subset of verbal problems was considered in this study. Hence, different results may have been obtained if a different subset of verbal problems had been used. Second, the potential influence of this factor may not be reflected in the data because little if any attention is given to the relationship between open sentences and verbal problems in the instructional program associated with the sample children's mathematics classes.

Interactions

In addition to the statistically significant main effects due to open sentence type and number size there was a significant interaction between these two factors. This interaction involved the extreme difficulty of the N - a = b type open sentences. Interpretation of this interaction may be summed up by saying that the N - a = b type open sentences were so difficult that the influence of number size was obscured or did not operate.

IMPLICATIONS

It was previously concluded that boys' and girls' performance are very similar when solving certain open sentences. Put another way, there were no data from this study which suggests that different instruction for boys and for girls is necessary or desirable.

Although the data indicate that one particular order of presentation effect did not exist it is essential that this result not be misinterpreted. That is, this conclusion is not related to instructional sequencing. In fact, the order in which open sentences of different kinds are presented in children's mathematics classes could very well affect their achievement in solving open sentences.

Although a full explanation of the differences in solving performance on the open sentence types is not possible at this time, the large differences in solving performance have instructional implications. If it is expected that children at this level should be able to solve equally well open sentences of the four types studied, then attention must be given to the $N - a = b$ type of open sentence. This attention may take many forms; for example, a new instructional approach may be needed, or more practice on this type of open sentence may be required.

If an objective of mathematics instruction at the third-grade level is "good" solving performance on open sentences with constants from the Two-digit domain, then current instruction must be reconsidered. That is if third-grade children are expected to solve open sentences with constants from the Two-digit domain as well as they solve open sentences with constants from the Basic Fact domain, then something different must take place in the children's learning experiences. For example, more explicit instruction on the relationship between open sentences such as $38 + N = 49$ and $7 + N = 16$ (i.e., sentences with the placeholder in the same position) may be desirable. Optimal procedures can only be determined after careful study.

The presence of an appropriate verbal problem did not seem to influence the solving of open sentences. However, this should not preclude exploiting the naturel ties between these two important mathematical topics. In fact, increased emphasis on these ties may be worth considering.

SUGGESTIONS FOR FUTURE RESEARCH

The results and conclusions reported in this paper need to be examined for validity with children of different mathematical backgrounds. The present study also needs to be extended to other open sentence types involving other operations and other number domains. For example, the operations of multiplication and division and the number domains composed of integers and rational numbers seem to be a plausible next step. Also, there is a need to examine other factors such as symmetric form (e.g., $N + a = b$ and $b = N + a$) across operations and number domains.

Investigations related to the research reported here where the opportunity to learn factor is held constant are also needed. In addition, a feasibility study involving the "effectiveness" of teaching a systematic method of solving open sentences of various types to third-grade children would be of interest.

REFERENCES

Finn, J. D. Multivariance: Fortran Program for Univariate and Multivariate Analysis of Variance and Covariance. Buffalo, N.Y.: Department of Educational Psychology, State University of New York at Buffalo, 1967.

Grouws, D. A. Differential Performance of Third Grade Children in Solving Open Sentences of Four Types. Unpublished doctoral dissertation, University of Wisconsin, 1971.

Suppes, P., Jerman, M., & Brian, D. Computer-assisted instruction: Stanford's 1965-66 arithmetic program. New York: Academic Press, 1968.

Weaver, J. F. Some Factors Associated with Pupil's Performance Levels on Simple Open Addition and Subtraction Sentences. Arithmetic Teacher, 1971, 18, 513-519.